1. Выполнить деление комплексных чисел:

1.1. $\frac{1+2i}{3+4i}=\frac{(1+2i)(3-4i)}{(3+4i)(3-4i)}=\frac{3-4i+6i-8i^2}{9-16i^2}=\frac{11+2i}{9+16}=\frac{11+2i}{25}=$

$=\frac{11}{25}+\frac{2}{25}i$;

1.2. $\frac{\sqrt{3}+\sqrt{2}i}{\sqrt{3}-\sqrt{2}i}=\frac{(\sqrt{3}+\sqrt{2}i)(\sqrt{3}+\sqrt{2}i)}{(\sqrt{3}-\sqrt{2}i)(\sqrt{3}+\sqrt{2}i)}=\frac{3+\sqrt{6}i+\sqrt{6}i+2i^2}{3-2i^2}=\frac{1+2\sqrt{6}i}{5}=$

$=\frac{1}{5}+\frac{2\sqrt{6}}{5}i$.

2. Вычислить пределы последовательностей:

2.1. $\lim\limits_{n\to\infty}\frac{2n+3}{3n+5}=\left\{\frac{\infty}{\infty}\right\}=\lim\limits_{n\to\infty}\frac{\frac{2n}{n}+\frac{3}{n}}{\frac{3n}{n}+\frac{5}{n}}=\lim\limits_{n\to\infty}\frac{2+\overbrace{\frac{3}{n}}^{\to 0}}{3+\underbrace{\frac{5}{n}}_{\to 0}}=\frac{2}{3}$.

2.2. $\lim\limits_{n\to\infty}\left(\sqrt{3n+1}-\sqrt{n+2}\right)=\lim\limits_{n\to\infty}\frac{(\sqrt{3n+1}-\sqrt{n+2})(\sqrt{3n+1}+\sqrt{n+2})}{\sqrt{3n+1}+\sqrt{n+2}}=$

$=\lim\limits_{n\to\infty}\frac{3n+1-n-2}{\sqrt{3n+1}+\sqrt{n+2}}=\lim\limits_{n\to\infty}\frac{2n-1}{\sqrt{3n+1}+\sqrt{n+2}}=\left\{\frac{\infty}{\infty}\right\}=\lim\limits_{n\to\infty}\frac{\frac{2n}{n}-\frac{1}{n}}{\frac{1}{n}\sqrt{3n+1}+\frac{1}{n}\sqrt{n+2}}=$

$=\lim\limits_{n\to\infty}\frac{2-\frac{1}{n}}{\sqrt{\frac{3n}{n^2}+\frac{1}{n^2}}+\sqrt{\frac{n}{n^2}+\frac{2}{n^2}}}=\lim\limits_{n\to\infty}\frac{2-\overbrace{\frac{1}{n}}^{\to 0}}{\sqrt{\underbrace{\frac{3}{n}}_{\to 0}+\underbrace{\frac{1}{n^2}}_{\to 0}}+\sqrt{\underbrace{\frac{1}{n}}_{\to 0}+\underbrace{\frac{2}{n^2}}_{\to 0}}}=\frac{2-0}{0+0}=\frac{2}{0}=\infty$.

3. Используя признаки Даламбера и Коши, исследовать сходимость рядов:

3.1. $\sum\limits_{n=1}^{\infty}\frac{n^3}{3^n}$

$a_n=\frac{n^3}{3^n}$, $a_{n+1}=\frac{(n+1)^3}{3^{n+1}}$

Воспользуемся признаком Даламбера: $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=$

$=\lim\limits_{n\to\infty}\left(\frac{(n+1)^3}{3^{n+1}}\cdot\frac{3^n}{n^3}\right)=\lim\limits_{n\to\infty}\frac{(n+1)^3}{3n^3}=\lim\limits_{n\to\infty}\frac{n^3+3n^2+3n+1}{3n^3}=$

$$=\left\{\frac{\infty}{\infty}\right\}=\lim_{n\to\infty}\frac{\frac{n^3}{n^3}+\frac{3n^2}{n^3}+\frac{3n}{n^3}+\frac{1}{n^3}}{\frac{3n^3}{n^3}}=\lim_{n\to\infty}\frac{1+\frac{3}{n}+\frac{3}{n^2}+\frac{1}{n^3}}{3}=$$

$=\frac{1}{3}<1$ – значит, ряд сходится.

3.2. $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^n$

Воспользуемся признаком Коши:

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{n}{2n+1}\right)^n}=\lim_{n\to\infty}\frac{n}{2n+1}=\left\{\frac{\infty}{\infty}\right\}=\lim_{n\to\infty}\frac{\frac{n}{n}}{\frac{2n}{n}+\frac{1}{n}}=$$

$=\lim_{n\to\infty}\frac{1}{2+\frac{1}{n}}=\frac{1}{2}<1$ – ряд сходится.

4. Найти производные сложных функций:

4.1. $y=\sin(\ln x)$;

$$y'=\cos(\ln x)\cdot\frac{1}{x}=\frac{\cos(\ln x)}{x}.$$

4.2. $y=\ln\sqrt[6]{x}$

$$y'=\frac{1}{\sqrt[6]{x}}\cdot\frac{1}{6}x^{-\frac{5}{6}}=\frac{1}{6\sqrt[6]{x}\cdot\sqrt[6]{x^5}}=\frac{1}{6x}.$$

5. Вычислить неопределённые интегралы:

5.1. $\int x^3\sin 2x\,dx=\left\{\begin{matrix}x^3=u;\ du=3x^2dx\\ \sin 2x dx=dv;\ v=-\frac{1}{2}\cos 2x\end{matrix}\right\}=$

$=-\frac{1}{2}x^3\cos 2x+\frac{3}{2}\int x^2\cos 2x\,dx=\left\{\begin{matrix}x^2=u;\ du=2x\,dx\\ \cos 2x=dv;\ v=\frac{1}{2}\sin 2x\end{matrix}\right\}=$

$=-\frac{1}{2}x^3\cos 2x+\frac{3}{4}x^2\sin 2x-\frac{3}{2}\int x\sin 2x\,dx=$

$$= \left\{\begin{matrix} x = u; \ du = dx. \\ \sin 2x\,dx = dv; \ v = -\frac{1}{2}\cos 2x \end{matrix}\right\} = -\frac{1}{2}x^3\cos 2x + \frac{3}{4}x^2\sin 2x +$$

$$+ \frac{3}{4}x\cos 2x - \frac{3}{4}\int \cos 2x\,dx = -\frac{1}{2}x^3\cos 2x + \frac{3}{4}x^2\sin 2x + \frac{3}{4}x\cos 2x -$$

$$- \frac{3}{8}\sin 2x + C.$$

5.2. $$\int x^2 2^x dx = \left\{\begin{matrix} x^2 = u; \ du = 2x\,dx \\ 2^x dx = dv; \ v = \frac{2^x}{\ln 2} \end{matrix}\right\} = \frac{2^x}{\ln 2}x^2 - \frac{2}{\ln 2}\int x\cdot 2^x dx =$$

$$= \left\{\begin{matrix} x = u; \ du = dx \\ 2^x dx = dv; \ v = \frac{2^x}{\ln 2} \end{matrix}\right\} = \frac{2^x}{\ln 2}\cdot x^2 - \frac{2}{\ln 2}\cdot\frac{x\cdot 2^x}{\ln 2} + \frac{2}{\ln^2 2}\int 2^x dx =$$

$$= \frac{2^x}{\ln 2}x^2 - \frac{2x\cdot 2^x}{\ln^2 2} + \frac{2}{\ln^2 2}\cdot\frac{2^x}{\ln 2} + C = \frac{2^x}{\ln 2}\left(x^2 - \frac{2x}{\ln 2} + \frac{2}{\ln^2 2}\right) + C.$$

6. Найти частные производные I и II порядков:

6.1. $$z = \frac{1}{x^3 y^4 \cos x} = x^{-3} y^{-4} (\cos x)^{-1};$$

$$\frac{\partial z}{\partial x} = -3x^{-4}y^{-4}(\cos x)^{-1} + x^{-3}y^{-4}\cdot(-1)(\cos x)^{-2}\cdot(-\sin x) = -\frac{3}{x^4y^4\cos x} + \frac{\sin x}{x^3y^4\cos^2 x};$$

$$\frac{\partial z}{\partial y} = -4x^{-3}y^{-5}(\cos x)^{-1} = -\frac{4}{x^3y^5\cos x};$$

$$\frac{\partial^2 z}{\partial x^2} = 12x^{-5}y^{-4}(\cos x)^{-1} - 3x^{-4}y^{-4}\cdot(-1)(\cos x)^{-2}\cdot(-\sin x) + \frac{\cos x\cdot x^3y^4\cos^2 x - \sin x \cdot (3x^2y^4\cos^2 x + x^3y^4\cdot 2\cos x(-\sin x))}{(x^3y^4\cos^2 x)^2} = \frac{12}{x^5y^4\cos x} - \frac{3\sin x}{x^4y^4\cos^2 x} +$$

$$+ \frac{x^3y^4\cos^3 x - 3x^2y^4\sin x\cos^2 x - x^3y^4\sin 2x}{x^6y^8\cos^4 x};$$

$$\frac{\partial^2 z}{\partial y^2} = -4x^{-3}(\cos x)^{-1}\cdot(-5y^{-6}) = \frac{20}{x^3y^6\cos x};$$

$$\frac{\partial^2 z}{\partial x\partial y} = -4y^{-5}\cdot(-3x^{-4})(\cos x)^{-1} - 4y^{-5}\cdot x^{-3}(\cos x)^{-2}\cdot \sin x =$$

$$= \frac{12}{x^4y^5\cos x} - \frac{4\sin x}{x^3y^5\cos^2 x};$$

$$\frac{\partial^2 z}{\partial y\partial x} = 12x^{-4}y^{-5}(\cos x)^{-1} - 4x^{-3}y^{-5}\cdot(-\cos x)^{-2}\cdot(-\sin x) =$$

$$= \frac{12}{x^4y^5\cos x} - \frac{4\sin x}{x^3y^5\cos^2 x}.$$

6.2. $z = x^y$

$$\frac{\partial z}{\partial x} = yx^{y-1};$$

$$\frac{\partial z}{\partial y} = x^y\ln x;$$

$$\frac{\partial^2 z}{\partial x^2} = y(y-1)x^{y-2};$$

$$\frac{\partial^2 z}{\partial y^2} = x^y\ln x\cdot\ln x = x^y\ln^2 x;$$

$$\frac{\partial^2 z}{\partial x\partial y} = x^{y-1} + yx^{y-1}\ln x = x^{y-1}(1+y\ln x);$$

$$\frac{\partial^2 z}{\partial y\partial x} = yx^{y-1}\ln x + x^y\cdot\frac{1}{x} = yx^{y-1}\ln x + x^{y-1} = x^{y-1}(y\ln x+1).$$

7. Найти сумму матриц:

7.1. $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 2 \\ 1 & 4 \end{pmatrix}$

$$A+B=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}+\begin{pmatrix}-1 & 2\\ 1 & 4\end{pmatrix}=\begin{pmatrix}1-1 & 2+2\\ 3+1 & 4+4\end{pmatrix}=\begin{pmatrix}0 & 4\\ 4 & 8\end{pmatrix}.$$

7.2. $A=\begin{pmatrix}-8 & -3\\ -2 & -6\end{pmatrix}$, $B=\begin{pmatrix}-1 & -1\\ -1 & -1\end{pmatrix}$

$$A+B=\begin{pmatrix}-8 & -3\\ -2 & -6\end{pmatrix}+\begin{pmatrix}-1 & -1\\ -1 & -1\end{pmatrix}=\begin{pmatrix}-8-1 & -3-1\\ -2-1 & -6-1\end{pmatrix}=\begin{pmatrix}-9 & -4\\ -3 & -7\end{pmatrix}.$$

8. Найти произведение матриц:

8.1. $A=\begin{pmatrix}0 & 1\\ -2 & 3\end{pmatrix}$, $B=\begin{pmatrix}2 & 5\\ 0 & 1\end{pmatrix}$

$$A\cdot B=\begin{pmatrix}0 & 1\\ -2 & 3\end{pmatrix}\cdot\begin{pmatrix}2 & 5\\ 0 & 1\end{pmatrix}=\begin{pmatrix}0\cdot2+1\cdot0 & 0\cdot5+1\cdot1\\ -2\cdot2+3\cdot0 & -2\cdot5+3\cdot1\end{pmatrix}=\begin{pmatrix}0 & 1\\ -4 & -7\end{pmatrix}.$$

8.2. $A=\begin{pmatrix}6 & 2\\ 3 & 8\end{pmatrix}$, $B=\begin{pmatrix}0 & -6\\ 5 & 7\end{pmatrix}$

$$A\cdot B=\begin{pmatrix}6 & 2\\ 3 & 8\end{pmatrix}\cdot\begin{pmatrix}0 & -6\\ 5 & 7\end{pmatrix}=\begin{pmatrix}6\cdot0+2\cdot5 & 6\cdot(-6)+2\cdot7\\ 3\cdot0+8\cdot5 & 3\cdot(-6)+8\cdot7\end{pmatrix}=\begin{pmatrix}10 & -22\\ 40 & 38\end{pmatrix}.$$

9. Найти определители матриц:

9.1. $\Delta_A=\begin{vmatrix}3 & 1\\ 2 & 3\end{vmatrix}=3\cdot3-2\cdot1=9-2=7.$

9.2. $\Delta_A=\begin{vmatrix}3 & 6\\ 5 & 7\end{vmatrix}=3\cdot7-5\cdot6=21-30=-9.$

10. Решить систему уравнений:

10.1. $+\begin{cases}7x+2y=15\\ x-2y=7\end{cases}$

$8x=22$; $x=\frac{22}{8}=\frac{11}{4}$; $\frac{11}{4}-2y=7$; $-2y=7-\frac{11}{4}$;

$-2y = \frac{28-11}{4}$; $-2y = \frac{17}{4}$; $y = -\frac{17}{8}$.

Ответ: $\left(\frac{11}{4}; -\frac{17}{8}\right)$.

10.2. $\begin{cases} 9x = 11y + 5 \\ 6y = 12x - 8 \end{cases}$ $\begin{cases} 9x - 11y = 5 \\ -12x + 6y = -8 \mid :2 \end{cases}$ $\begin{cases} 9x - 11y = 5 \mid \cdot \frac{2}{3} \\ -6x + 3y = -4 \end{cases}$

$+\begin{cases} 6x - \frac{22}{3}y = \frac{10}{3} \\ -6x + 3y = -4 \end{cases}$

$\left(3 - \frac{22}{3}\right)y = \frac{10}{3} - 4$; $\frac{9-22}{3}y = \frac{10-12}{3}$; $-\frac{13}{3}y = -\frac{2}{3}$;

$y = -\frac{2}{3} : \left(-\frac{13}{3}\right) = -\frac{2}{3} \cdot \left(-\frac{3}{13}\right) = \frac{2}{13}$;

$-6x + 3 \cdot \frac{2}{13} = -4$; $-6x = -4 - \frac{6}{13}$; $-6x = \frac{-52-6}{13}$;

$-6x = \frac{-58}{13}$; $x = -\frac{58}{13} : (-6) = -\frac{58}{13} \cdot \left(-\frac{1}{6}\right) = \frac{29}{39}$.

Ответ: $\left(\frac{29}{39}; \frac{2}{13}\right)$.

11. Для заданных векторов найти смешанное произведение $[\bar{a} \times \bar{b}] \cdot \bar{c}$:

11.1. $\bar{a}(1; -2; 1)$, $\bar{b}(2; 1; -2)$, $\bar{c}(1; 1; 1)$

$[\bar{a} \times \bar{b}] \cdot \bar{c} = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix} = 1 \cdot 1 \cdot 1 + 2 \cdot 1 \cdot 1 + 1 \cdot (-2) \cdot (-2) - 1 \cdot 1 \cdot 1 - 1 \cdot 1 \cdot (-2) -$

$-2 \cdot (-2) \cdot 1 = 1 + 2 + 4 - 1 + 2 + 4 = 12$.

11.2. $\bar{a}(1;1;2)$, $\bar{b}(1;-1;3)$, $\bar{c}(-2;-2;2)$

$$[\bar{a} \times \bar{b}] \cdot \bar{c} = \begin{vmatrix} 1 & 1 & 2 \\ 1 & -1 & 3 \\ -2 & -2 & 2 \end{vmatrix} = 1 \cdot (-1) \cdot 2 + 1 \cdot 2 \cdot (-2) + 1 \cdot 3(-2) - 2 \cdot (-1) \cdot (-2) -$$

$$-1 \cdot 3 \cdot (-2) - 1 \cdot 1 \cdot 2 = -2 - 4 - 6 - 4 + 6 - 2 = -12.$$